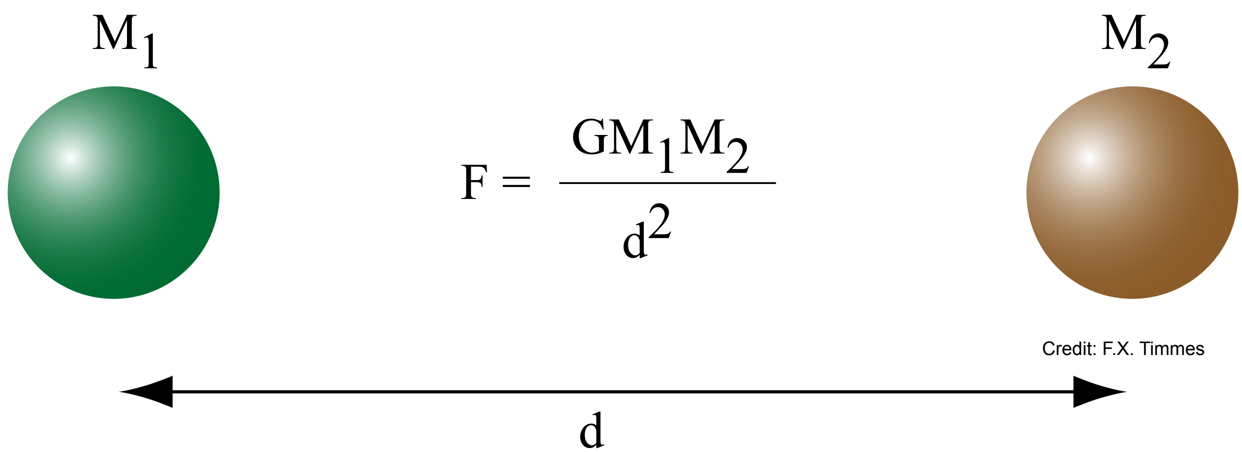
Equipped with his five senses, man explores
the universe around him and calls the adventure Science.
Edwin Hubble

The Gravity Of The Situation …

Hi AST111! Today we're going to cover how Newton's law of gravity explains and extends Kepler's laws.

What determines the strength of gravity? Well, in this hypothesis that Newton made up, the gravitational force between any two objects is equal to some universal constant we call G. It's just like the speed of light, or Planck's constant. G is a fundamental constant of nature, G. It's just a number.

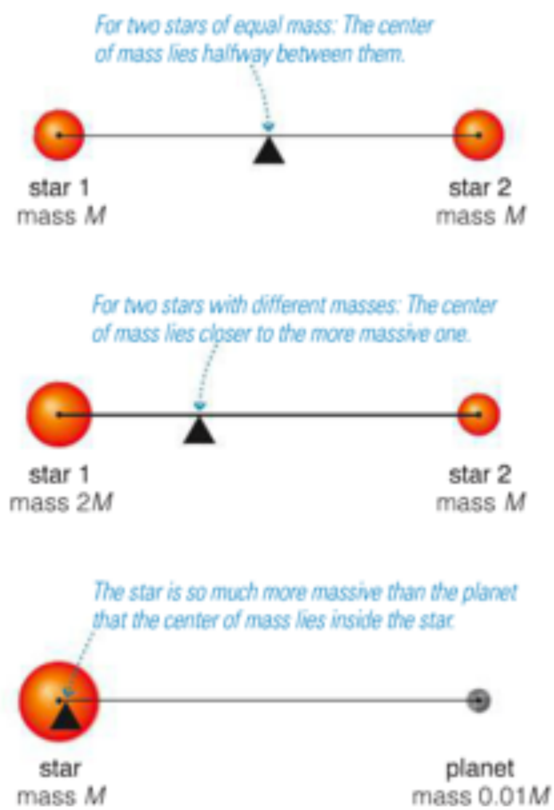


Credit: F.X. Timmes

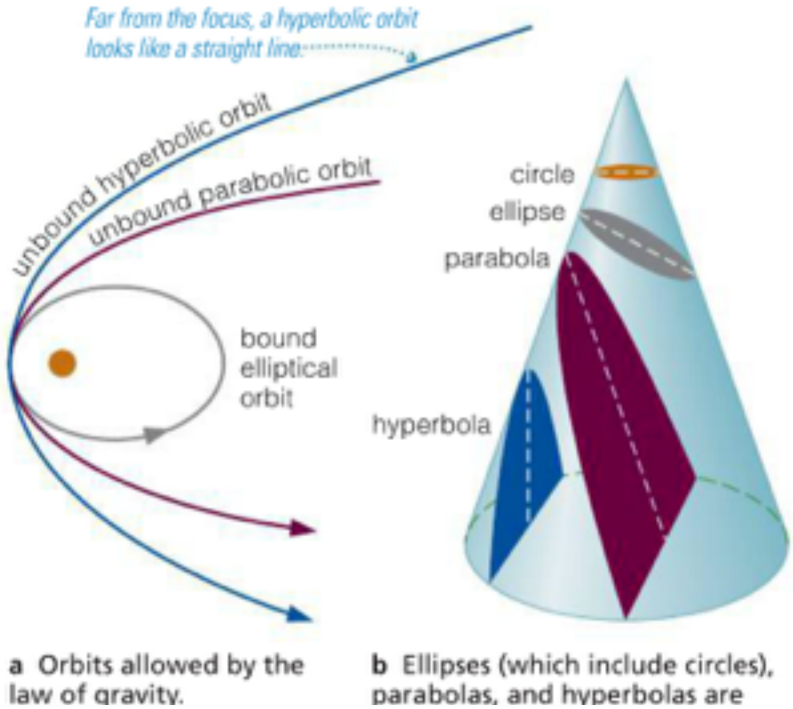
The important part is the force of gravity is proportional to the product of their masses, so mass #1 times mass #2 and divided by their separation squared. So it's a $1/r^2$ squared law of gravity. If you double the masses, one of the masses gets twice as massive, and the force gets twice as big. Why? Because the mass is in the numerator, and you get increased mass by a factor of two, so the force doubles. If you double the distance, 2 squared is 4, and so the force decreases by a factor of 4, or said another way, the force is 1/4 as strong because the distance squared is in the denominator. OK?

So this is Newton's hypothesis of how gravity behaves. It's a very accurate description of gravity, when the objects are not too massive and small. If the objects start getting particularly compact; for example, around a neutron star or black hole, then one has to use Einstein's general theory of relativity. But Einstein's general theory of relativity encompasses Newton's law, so that when you get far away from a deep gravitational well, Einstein's field equations reduce to the classical Newtonian law of gravity.

Newton’s gravity explains and extends Kepler's laws in four ways. First, he showed that two objects don't orbit each other. They each orbit about a common center of mass.



The image below shows, if you have two stars of equal mass, or two planets of equal mass, your lever point — the center of mass— is given by the triangle there, and they will both orbit around that common center of mass. If one of the stars is a lot more massive, that balance point triangle moves over closer to the more massive object. That balance point, if you like, is closer, so they both orbit around that center of mass. If the situation reminds you of a see-saw on a playground, you are exactly right! And if one object is massive enough, then the center of mass actually moves inside the body, that's fine, it's just a balance point, and they both orbit around that common center of mass.



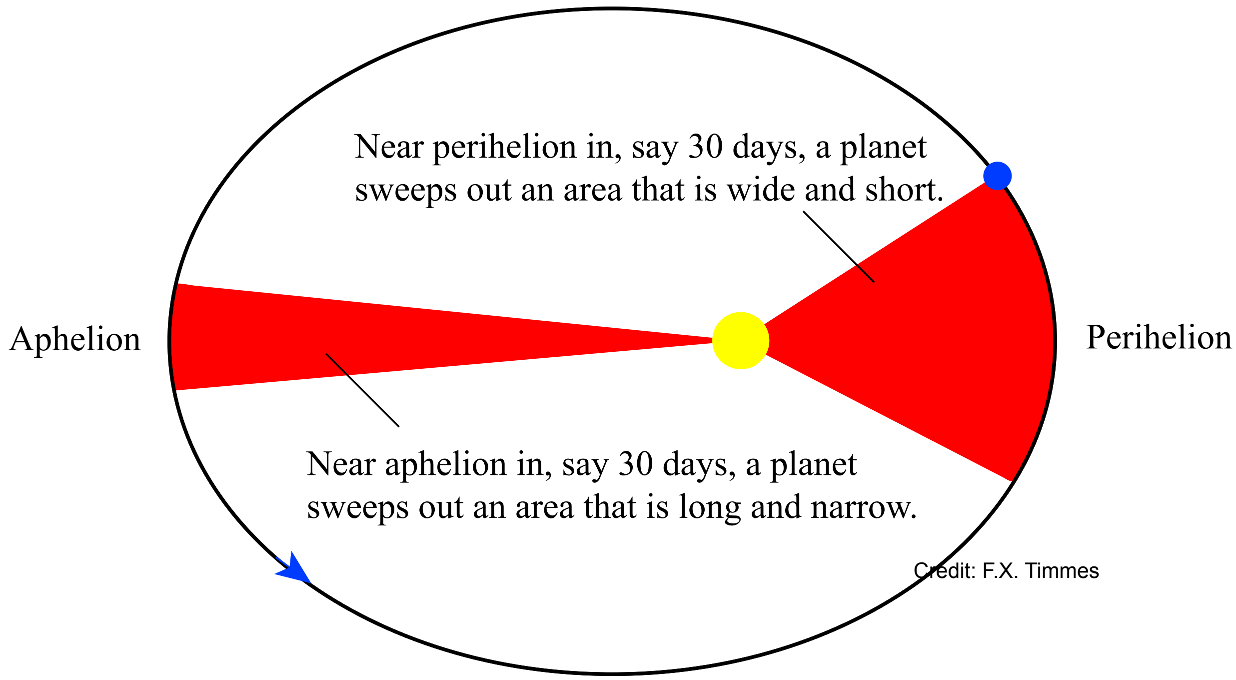
Credit: F.X. Timmes

For example, in the Earth and Moon system, the common center of mass is actually inside the Earth. That's fine. They both do their orbital dance, around their common center of mass. Thus it's incorrect to say that the Moon orbits the Earth. That's not true. The Moon and the Earth both orbit each other around their common center of mass. This is a concept that you should get down.

Kepler showed that the orbits of the planets were ellipses, with the Sun at one foci. Newton extended this to show there were other kinds of orbits, not just elliptical orbits. You can have things like parabolic orbits or hyperbolic orbits. You can have circular orbits, of course, but that is just a special case of the ellipse. These are all geometric objects as shown in the image above.

Elliptical and circular orbits are “bound orbit.” The other two types of orbits, parabolic or hyperbolic, are “unbound orbits.” Comets and asteroids come in typically on hyperbolic or parabolic orbits. They zip around the Sun once and they go off, never to be seen from again, because it's not gravitationally bound orbit.

Take a right circular cone and a slicing plane. How you slice the cone determines what kind of geometric cross-section you get. If you cut the cone horizontally, then you get a circle. If you cut the cone at an angle, you get an ellipse. If you keep increasing the angle of the cutting plane, eventually you'll run into the bottom of the cone, and that gives you a parabolic orbit. And if you cut the cone vertically you get a hyperbola. So there you have four different types of orbits and their relationship to geometry.



The areas swept out in any 30 day period are equal.

Third, Newton's law of gravity shows that Kepler's first two laws, ellipses and equal area in equal time, are a consequence of the conservation of angular momentum (mass × speed × distance = constant). For example, near perihelion the distance to the Sun is smaller. And so in order to keep the mass times the speed times the distance the same, the conservation of angular momentum, the plane must move faster. Conversely, when you're on aphelion, when you're farther from the sun, the distance is large, and so in order to keep mass times speed times distance the same, the speed must decrease. Kepler's second law of equal area in equal time is just a consequence of conservation of angular momentum.

$$\mathrm{P}^2 = \frac{4\pi^2}{G\left(\mathrm{M}_{\mathrm{sun}} + \mathrm{M}_{\mathrm{planet}}\right)}\mathrm{a}^3$$

And finally, we've learned Kepler's law as $P^2 = a^3$. The period *P* is the time it takes to go around and the semi-major axis *a* of the ellipse, or as we've learned *a* is also the average orbital distances. That's very useful. So P squared is a cubed.

Newton showed that $P^2 = a^3$ was actually a special case of the equation shown above. You can see the P^2 and a^3 parts in the formula above, but now we have the factor of 4 pi^2 divided by G times the sum of the masses.

Wow, the masses! Newton's version of Kepler's third law allows us to find the masses of orbiting objects if we know the periods and distances.The importance of this formula cannot be overstated. If you ask, how much mass does an object on the celestial sphere have? In general, we don't know. The only way that we know how to weigh anything directly is if one object orbits another object a binary system. We measure the period P. That's easy, just break out your clock, watch the thing go around. Then we measure the average separation, sometimes directly sometimes indirectly. Then you use the formula above to "weigh" the mass of the system.
Cool. Peace out.

Bye Bye!